

Many-polaron system in Fractional Dimensional space within the Plasmon Pole Approximation

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(Dated: October 21, 2014)

Abstract

The polaron binding energy and effective mass in a degenerate polar gas is calculated in the fractional-dimensional approach under plasmon pole approximation. The effect of carrier densities on the static and dynamic screening correction of the electron-phonon interaction and electron-electron interaction to the polaronic properties is calculated from electron self-energies within the second-order perturbation method. The Hubbard local field factor has been used for the static screening correction in the polaronic properties. We found that polaronic properties decrease with increase with carrier density and dimensionality of the system.

PACS numbers: 73.20Dx, 85.60.Gz, 79.40.+z

I. INTRODUCTION

When an electron in the bottom of the conduction band of a polar semiconductor moves, its Coulomb field displaces the positive and negative ions with respect to each other producing polarization field. The electron with the associated phonon cloud is known as a polaron.

The density oscillation in a medium creates plasmons. The interaction between an electron and plasmons can be described by Fröhlich Hamiltonian. The interaction of the electron with plasmon cloud is termed as the plasmapolaron. The binding energy is shifted and electron mass is enhanced.

The Fröhlich polaron results when the interaction between electron and longitudinal optic (LO) phonon is described in the Fröhlich form characterized by the dimensionless coupling constant α . The polaron in an undoped polar material calculated with the unscreened electron-phonon interaction is termed as a single-polaron.

In the weak coupling limit ($\alpha < 1$), the perturbation theory is applied to calculate ground state energies and effective masses of the single-polaron in three-dimensional (3D) polar materials[4], two-dimensional (2D) quantum well (QW) structures[?] and one-dimensional (1D) quantum wire structures[6]. The polaron properties were also calculated in the multidimensional space (nD) where the space has an integer dimension n . In the nD space, the expressions for the ground state energy and effective mass were correctly expressed in terms of n in the weak[?], intermediate[?] and strong coupling[?] limits. Since the Coulomb interaction in Fourier method is divergent in the 1D systems, the multidimensional method fails to calculate polaron properties in the quantum wire structures. Therefore, the multidimensional space method finds the polaron properties in 2D and 3D systems quite accurately. When the width of the QW is extremely narrow and the barrier potential that causes the in-plane confinement is infinite, the system is purely 2D. The dimension increases monotonically with the increase of the well width and the infinitely wide well exhibits the three-dimensional (3D) behaviour. In a finite QW with narrow well width, the electron envelope function spreads into the barrier region partially restoring the 3D characteristics of the system. The properties of the QW are determined by the parameters of the barrier materials. In the QW with large well width, the properties of the polaron are calculated taking the bulk values of the well material. This has been demonstrated in the calculation

of exciton binding energy[?] and polaron properties as a function of well width[?]. Consequently, the QW with finite well width and barrier height shows fractional dimensional behaviour (βD) where $2 < \beta < 3$. The anisotropic interactions in an anisotropic solid are treated as ones in an isotropic fractional dimensional space, where the dimension is determined by the degree of anisotropy. Thus only a single parameter known as the degree of dimensionality β is needed to describe the system. The fractional dimensional space is not a vector space and the coordinates in this space are termed as *pseudocoordinates*[7].

The fractional dimensional space method has been applied to study exciton, biexciton, magnetoexciton and impurity states. The single-polaron binding energy and effective mass in the weak coupling limit have been derived in rectangular and parabolic QWs. The results are found to increase with decreasing confinement.

When the carrier density is high, the electron-phonon interaction is dynamically screened by the frequency dependent dielectric function and the resulting polaron is termed as the many-polaron. However, for systems with much larger plasma frequency than the LO phonon frequency, the static dielectric function can also be a good approximation. This happens when the carrier density is very high in the system. The properties of many-polaron in the doped ZnS was studied by da Costa and Studart[9] including exchange-correlation effects beyond RPA and found that the ground state energies and effective masses are influenced by the exchange-correlation effects.

The static properties such as the binding energy and optical conductivity have been studied by including the dynamical screening method. The effective mass of the polaron has been found from the optical conductivity. The results are markedly different from those calculated using the static dielectric function[16–18].

In the present work we study the many-polaron system by the static and dynamically screened electron-phonon interaction and electron-electron interaction.

II. POLARON BINDING ENERGY AND EFFECTIVE MASS DUE TO STATIC SCREENING EFFECT

In the second-order perturbation method, the polaron binding energy is calculated from the electron self-energy due to electron-phonon interaction as[1]

$$E_{\beta D} = -\Sigma_{\beta D}(\mathbf{k}, \xi_k)|_{\mathbf{k}=0}, \quad (1)$$

where ξ_k is the electron energy with parabolic band dispersion ($\hbar^2 k^2/2m_b$). The effective mass in the same method is defined as

$$\frac{m_b}{m^*} = 1 + \left(\frac{m_b}{\hbar^2} \right) \frac{\partial^2 \Sigma_{\beta D}(\mathbf{k}, \xi_k)}{\partial k^2} \Big|_{\mathbf{k}=0} \quad (2)$$

where m_b is the band electron mass.

The leading Feynman diagram contribution to the electron self energy due the electron-phonon interaction is given by

$$\Sigma_d(p, \iota k_m) = -\frac{1}{\beta} \sum_{m,q} V_{eff}(q, \omega) G^0(\iota \omega_n, \xi_{p+q})$$

Here

$$V_{eff}(q, \omega) = \frac{M_d(q, \omega)^2}{\epsilon_d(q, \omega)^2} D_d(q, \omega)$$

is the effective potential for electron-phonon interaction.

$D_d(q, \omega)$ is the renormalized phonon propagator given by

$$D_d(q, \omega) = \frac{D_0(q, \omega)}{1 - |M_d(q)|^2 D_0(q, \omega) \epsilon_d(q, \omega)}$$

$$D_0(q, \omega) = \frac{2\omega_{LO}}{\omega^2 - \omega_{LO}^2}$$

. is the unperturbed phonon propagator.

The free electron Green function $G^0(\iota \omega_n, \xi_{(p+q)})$ is given as

$$G^0(\iota \omega_n, \xi_{(p+q)}) = \frac{1}{\iota p_m + i\hbar \omega_n - \xi_{p+q}}$$

Using eq(), eq() in eq() we get the self energy equation becomes

$$\Sigma(\mathbf{k}, \iota k_m) = -\frac{1}{\beta} \sum_{\mathbf{q}} \frac{|M(q)|^2}{\epsilon(q, rs)^2} \sum_{\iota \omega_n = -\infty}^{\infty} \frac{2\omega_{LO}}{(\iota \omega_n)^2 - (\omega_{LO})^2} \times \frac{1}{\iota p_m + i\hbar \omega_n - \xi_{p+q}}$$

Using Matsubara frequency summation the electron-phonon self energy becomes

$$\Sigma(\mathbf{k}, \iota k_m) = -\frac{1}{\beta} \sum_{\mathbf{q}} \frac{|M(q)|^2}{\epsilon(q, rs)^2} \times \left[\frac{1 + n_B(\omega_{LO} - n_F \xi_{p+q})}{\iota p_m - \hbar \omega_{LO} - \xi_{p+q}} + \frac{n_B(\omega_{LO} + n_F \xi_{p+q})}{\iota p_m + \hbar \omega_{LO} - \xi_{p+q}} \right]$$

At zero temperature both n_F and n_B vanish and the self energy equation becomes

$$\Sigma(\mathbf{k}, ik_m) = -\frac{1}{\beta} \sum_{\mathbf{q}} \frac{|M(q)|^2}{\epsilon(q, rs)^2} \times \left[\frac{1}{\xi_p - \xi_{p+q} - \hbar\omega_{LO}} \right]$$

The electron-phonon interaction term is expressed as

$$M_{\beta D}(q) = -i\hbar\omega_{LO} \left(\frac{(4\pi)^{\frac{\beta-1}{2}} \Gamma\left(\frac{\beta-1}{2}\right) R_p \alpha}{q^{\beta-1} \Omega_{\beta}} \right)^{\frac{1}{2}}, \quad (3)$$

where Γ is the Euler-gamma function and $R_p = \sqrt{\hbar/2m_b\omega_{LO}}$ is the polaron radius. The dimensionless coupling constant α is defined as

$$\alpha = \frac{e^2}{2\hbar\omega_{LO}R_p} \left(\frac{1}{\epsilon_{\infty}} - \frac{1}{\epsilon_0} \right), \quad (4)$$

where ϵ_0 and ϵ_{∞} are the static and high-frequency dielectric constants, respectively. The static dielectric function including the local-field factor is defined as

$$\epsilon_{\beta D}(q, rs) = \epsilon_{\infty} \left[\frac{1 - [1 - G_{\beta D}(q, rs)] V_{\beta D}(q) \chi_{\beta D}(q, rs)}{1 + G_{\beta D}(q, rs) V_{\beta}(q) \chi_{\beta D}(q, rs)} \right], \quad (5)$$

where $G_{\beta D}(q, rs)$ is the Hubbard local-field-factor given by

$$G_{\beta D}(q, rs) = \frac{1}{2} \frac{q^{\beta-1}}{(q^2 + k_F^2)^{\frac{\beta-1}{2}}}. \quad (6)$$

The dimensionless density parameter rs is given by

$$k_F r_s a_B = \left[2^{\beta-1} \Gamma^2 \left(1 + \frac{\beta}{2} \right) \right]^{\frac{1}{\beta}} \quad (7)$$

where a_B is Bohr atomic radius and k_F is Fermi wave vector.

The irreducible polarizability function $\chi_{\beta D}(q, rs)$ is defined as

$$\chi_{\beta D}^0(q, rs) = \sum_{\mathbf{k}} \frac{n_F(\mathbf{k} + \mathbf{q}) - n_F(\mathbf{k})}{\xi_{\mathbf{k}+\mathbf{q}} - \xi_{\mathbf{k}}}. \quad (8)$$

In the fractional dimensional method the sum over q is transferred to integration as

$$\sum_{\mathbf{q}} \cdots = \frac{V}{(2\pi)^d} \frac{2\pi^{\frac{d-1}{2}}}{\Gamma(\frac{d-1}{2})} \int dq q^{d-1} \int_0^{\pi} (\sin \theta)^{d-2} \cdots$$

Using Eq.(10) in Eq. (9), we find

$$\chi_{\beta D}^0(q, rs) = -\frac{2^{3-\beta} m_b k_F^{\beta}}{\pi^{\frac{\beta-1}{2}} \hbar^2 \beta q^2 \Gamma\left(\frac{\beta-1}{2}\right)} \int_0^{\pi} {}_2F_1\left(1, \frac{\beta}{2}; \frac{\beta+2}{2}; \frac{4k_F^2 \cos^2 \theta}{q^2}\right) \sin^{\beta-2} \theta d\theta, \quad (9)$$

where ${}_2F_1$ is the Gauss hypergeometric function.

The Fourier transform of e^2/r in the fractional dimensional space is obtained as

$$V_{\beta D}(q) = \frac{(4\pi)^{\frac{\beta-1}{2}} e^2 \Gamma\left(\frac{\beta-1}{2}\right)}{q^{\beta-1}} \quad (10)$$

Using Eqs. (6), (7), (10) and (12) in Eq. (1), the binding energy is obtained as

$$E_{\beta D} = -\alpha \hbar \omega_{LO} \frac{\Gamma\left(\frac{\beta}{2}\right)}{\sqrt{\pi} \Gamma\left(\frac{\beta}{2}\right)} \int_0^\infty \frac{dq}{\epsilon_{\beta}(q, r_s)(q^2 + 1)}. \quad (11)$$

Similarly the effective mass in Eq.(2) can be obtained as

$$\frac{m_b}{m^*} = 1 - 4\alpha \frac{\Gamma\left(\frac{\beta-1}{2}\right)}{\sqrt{\pi} \beta \Gamma\left(\frac{\beta}{2}\right)} \int_0^\infty \frac{q^2 dq}{\epsilon_{\beta D}^2(q, r_s)(q^2 + 1)^3}. \quad (12)$$

For nondegenerate systems($rs \rightarrow \infty$), $\epsilon_{\beta D} = 1$. The integrals in above Eqs(11) and (12) can be analytically evaluated. Now The binding energy is derived as

$$E_{\beta D} = -\frac{1}{2} \alpha \hbar \omega_{LO} \frac{\sqrt{\pi} \Gamma\left(\frac{\beta-1}{2}\right)}{\Gamma\left(\frac{\beta}{2}\right)} \quad (13)$$

and the effective mass is given by

$$\frac{m_b}{m^*} = 1 + \frac{1}{4} \alpha \frac{\sqrt{\pi} \Gamma\left(\frac{\beta-1}{2}\right)}{\beta \Gamma\left(\frac{\beta}{2}\right)}, \quad (14)$$

III. RESULT AND DISCUSSION

we have taken the several parameters of GaAs to calculate polaron properties. The band mass $m_b = 0.067m_0$, where m_0 is the electron mass, $\epsilon_\infty = 13.18$ and $\epsilon_0 = 10.89$. The value of $\alpha = 0.03$ which is appropriate for calculating polaron properties in the weak coupling limit. The LO phonon energy ($\hbar\omega_{LO}$) is taken as 36.25 meV.

The polaron binding energy and effective masses due to static screening correction of electron-phonon interaction is calculated for several r_s values for dimensions $\beta = 2, 2.5$ and 3

are shown in Table I. Both binding energies and effective masses are found to increase with the increasing r_s . This suggests that the polaron properties decrease with the increasing carrier density. This results due to screening of the electron-phonon interaction.

The polaron properties also decrease as the dimension decreases. As the dimension of the system decreases, the system becomes more confined. The confinement of the system decreases the physical properties.

Although the static Hubbard's local-field-factor for the screening of the electron-phonon interaction correctly predicts r_s and β dependence of the physical properties, it does not include the exchange and correlation effects properly. It is much higher than the dynamic local field factor as the later correctly includes the exchange and correlation effects. Therefore it is required that we calculate the dynamic local field factor using quantum version of the STLS method and screen the electron-phonon interaction term. Such a work is in progress in our group.

IV. POLARON BINDING ENERGY AND EFFECTIVE MASS DUE TO DYNAMIC SCREENING EFFECT

Conventionally the Raleigh-Schrödinger (RS) and Tamm-Dancoff approximation to the Brillouin-Wigner (TD-BW) perturbation theories are employed to calculate ground state energies and effective masses of the single-polaron state[1]. The effective masses calculated in the RS perturbation theory were more consistent than the TD-BW methods. This scenario did not change when the effective masses of the many-polaron system were calculated in bulk semiconductors. We have therefore taken the RS perturbation theory to calculate binding energies and effective masses of fractional dimensional many-polaron.

In the RS perturbation method, the polaron binding energy is calculated from the electron self-energy due to electron-phonon interaction as

$$E_{pol} = -\Sigma(\mathbf{k}, \xi_{\mathbf{k}})|_{\mathbf{k}=0}, \quad (15)$$

where $\xi_{\mathbf{k}} = \hbar^2 k^2 / 2m_b$ with m_b being the band mass is the parabolic band dispersion. The nonparabolicity of the energy dispersion is ignored. The effective mass in the same method is defined as

$$\frac{m_b}{m^*} = 1 + \left(\frac{m_b}{\hbar^2} \right) \frac{\partial^2 \Sigma(\mathbf{k}, \xi_{\mathbf{k}})}{\partial k^2} \bigg|_{\mathbf{k}=0}, \quad (16)$$

The electron-electron and electron-phonon interactions in the form

$$V_d(q, \omega) = \frac{v_d(q)}{\epsilon_d(q, \omega)} + \frac{M_d(q)^2}{\epsilon_d(q, \omega)^2} D_d(q, \omega) \quad (17)$$

where $v_d(q)$ is the Fourier transform of the Coulomb potential, $\epsilon_d(q, \omega)$ is the dielectric function, $|M_d(q)|^2$ is the electron-phonon interaction term and $D_d(q, \omega)$ is the phonon renormalized Green's function.

The strength of electron-phonon interaction $M_d(q)$ of a material depends on the optical properties of that material. In an anisotropic low dimensional structure, there are confined, half-space and interface phonon modes as a consequence of the presence of heterointerfaces. Consequently, a rigorous treatment of the electron-phonon interaction in semiconductor heterostructures requires the consideration of all these modes[? ?]. Since the anisotropic Euclidean space in low dimensional structures are treated isotropic in FD space, the electron phonon interaction is treated similar to bulk modes.

The renormalized phonon propagator is defined as

$$D_d(q, \omega) = \frac{2\omega_{LO}}{\omega^2 - \omega_{LO}^2 - |M_d(q)|^2 \omega_{LO} \chi_d^0(q, \omega) / \epsilon_d(q, \omega)}, \quad (18)$$

where $\chi^0(q, \omega)$ is the irreducible polarizability function. In the single-pole plasmon approximation, $\chi^0(q, \omega)$ is defined as

$$\chi_d^0(q, \omega) = \frac{nq^2}{m_b} \frac{1}{\omega^2 - \Omega_d^2(q) + \omega_d^2(q)} \quad (19)$$

In the PPA method $\epsilon(q, \omega)$ is expressed as

$$\frac{1}{\epsilon_d(q, \omega)} = \frac{1}{\epsilon_\infty} \left[1 + \frac{\omega_d^2(q)}{\omega^2 - \Omega_d^2(q)} \right] \quad (20)$$

The plasmon frequency in the long-wavelength limit within the PPA method is given as[?]

$$\Omega_d^2(q) = \omega_d^2(q) + q^2 v_F^2 \left[\frac{1}{d} - \frac{(7-d)\Gamma(\frac{d}{2})}{4(d+2)\pi^{\frac{1}{2}}\Gamma(\frac{d+3}{2})} \left(\frac{q_{TF}}{k_F} \right)^{d-1} \right] \quad (21)$$

In this equation v_F is the Fermi velocity, $v_F = \hbar k_F / m^*$ with k_F being the Fermi momentum.

The Thomas-Fermi momentum is defined as

$$q_{TF}^{d-1} = \frac{2m_b e^2 k_F^{d-2}}{\sqrt{\pi} \epsilon_\infty \hbar^2} \left[1 + \left\{ \Gamma \left(1 + \frac{d}{2} \right) \right\}^2 \right]^{\frac{1}{d}} \quad (22)$$

Using Eq.() and (), $D(q, \omega)$ is defined as

$$D_d(q, \omega) = \frac{2\omega_{LO}[\omega^2 - \Omega_d^2(q)]}{\hbar[\omega^2 - \omega_+^2(q)][\omega^2 - \omega_-^2(q)]} \quad (23)$$

where

$$\omega_{\pm}^2(q) = \frac{1}{2}[\omega_{LO}^2 + \Omega_d^2(q)] \pm \frac{1}{2}\sqrt{[\omega_{LO}^2 - \Omega_d^2(q)]^2 + 4\omega_d^2(q)(\omega_{LO}^2 - \omega_{TO}^2)} \quad (24)$$

In the presence of free electrons the LO-phonon frequency is renormalized due to plasmon-phonon coupling giving rise to two coupled modes with frequencies $\omega_+(q)$ and $\omega_-(q)$ which lie close to the uncoupled modes away from the mode-coupling region.

The leading-order contribution to the electron self-energy due to the electron-phonon interaction is given by

$$\Sigma_d(\mathbf{k}, ik_m) = -\frac{1}{\beta} \sum_{\mathbf{q}} \sum_{i\omega_n} \frac{|M(q)|^2}{\epsilon(q, i\omega_n)^2} D(q, i\omega_n) G^0(\mathbf{k} + \mathbf{q}, ik_m + i\hbar\omega_n), \quad (25)$$

where $G^0(\mathbf{k} + \mathbf{q}, ik_m + i\hbar\omega_n)$ is the free electron Green function and ip_m and $i\omega_n$ are the standard Fermi and Bose imaginary frequencies, respectively, in the Matsubara's formalism. The free electron Green function $G^0(i\omega_n, \xi_{(p+q)})$ is given as

$$G^0(k + q, i\omega_n) = \frac{1}{i\omega_n + i\hbar\omega_n - \xi_{k+q}}$$

This is the one-phonon self-energy term which is justified in the weak coupling limit. The purely electronic dielectric function $\epsilon(q, i\omega_n)$ contains all informations about screening. Substituting Eqs(20), (23) and (26) in Eq.(25), we find

$$\begin{aligned} \Sigma(\mathbf{k}, ik_m) = & -\frac{1}{\beta} \sum_{\mathbf{q}} |M(q)|^2 \sum_{i\omega_n=-\infty}^{\infty} \frac{2\omega_{LO}[(i\omega_n)^2 - \Omega^2(q) + \omega_p^2(q)]^2}{[(i\omega_n)^2 - \omega_+^2(q)][(\omega_n)^2 - \omega_-^2(q)][(i\omega_n)^2 - \Omega^2(q)]} \\ & \times \frac{1}{ik_m + i\hbar\omega_n - \xi_{\mathbf{k}+\mathbf{q}}} \end{aligned} \quad (26)$$

The classical method outlined in Ref.9 is followed to carry out the frequency summation. First of all we form the triplet $(\omega_+\omega_-\Omega)$ and indicate it as $(\omega_1\omega_2\omega_3)$. The $i\omega_n$ summation is converted into an integral as $I = \int dz f(z)n_b(z)/2\pi i$ where $z = i\omega_n$ and n_B is the Bose distribution function. Using the contour integration method the poles and residues of $n_B(z)$ and $f(z)$ are determined. The integral I is then found by adding all residues. In the Jordan's lemma $I = 0$ when $R \rightarrow \infty$. Finally in the analytic continuum $ip_m = \xi_{\mathbf{k}} + i\delta$, a

final expression for the self-energy has been derived as

$$\Sigma(\mathbf{k}, \xi_{\mathbf{k}}) = \sum_{\mathbf{q}} |M_d(q)|^2 \sum_{ijk} \epsilon_{ijk} \left[\frac{F_{ijk}(q)n_B(\omega_i) + G_{ijk}(\mathbf{p}, \mathbf{q})n_F(\xi_{\mathbf{p}+\mathbf{q}})}{\xi_{\mathbf{p}} - \xi_{\mathbf{p}+\mathbf{q}} + \hbar\omega_i + i\delta} + \frac{F_{ijk}(q)[n_B(\omega_i) + 1] - G_{ijk}(\mathbf{p}, \mathbf{q})n_F(\xi_{\mathbf{p}+\mathbf{q}})}{\xi_{\mathbf{p}} - \xi_{\mathbf{p}+\mathbf{q}} - \hbar\omega_i + i\delta} \right], \quad (27)$$

where ϵ_{ijk} is defined as $\epsilon_{123} = \epsilon_{231} = \epsilon_{312} = 1$ and all other combinations ijk are zero, $F_{ijk}(q)$ is obtained as

$$F_{ijk}(q) = \frac{(\omega_i^2(q) - \Omega^2(q) + \omega_p^2(q))^2 \omega_{LO}}{\omega_i(q)[\omega_i^2(q) - \omega_j^2(q)][\omega_i^2(q) - \omega_k^2(q)]} \quad (28)$$

and $G_{ijk}(\mathbf{k}, \mathbf{q})$ is derived as

$$G_{ijk}(\mathbf{k}, \mathbf{q}) = \frac{\omega_{LO}[(\xi_{\mathbf{k}+\mathbf{q}} - \xi_{\mathbf{k}})^2 - \hbar^2\Omega_d^2(q) + \hbar^2\omega_d^2(q)]^2}{\hbar\omega_i(q)[(\xi_{\mathbf{k}+\mathbf{q}} - \xi_{\mathbf{k}})^2 - \hbar^2\omega_j^2(q)][(\xi_{\mathbf{k}+\mathbf{q}} - \xi_{\mathbf{k}})^2 - \hbar^2\omega_k^2(q)]} \quad (29)$$

At zero-temperature, both n_F and n_B vanish and the expression for the self-energy is obtained as

$$\Sigma(\mathbf{k}) = - \sum_{\mathbf{q}} |M_d(q)|^2 \left[\frac{F_{123}(q)}{\xi_{\mathbf{k}} - \xi_{\mathbf{k}+\mathbf{q}} - \hbar\omega_+(q)} + \frac{F_{231}(q)}{\xi_{\mathbf{k}} - \xi_{\mathbf{k}+\mathbf{q}} - \hbar\omega_-(q)} + \frac{F_{312}(q)}{\xi_{\mathbf{k}} - \xi_{\mathbf{k}+\mathbf{q}} - \hbar\Omega(q)} \right] \quad (30)$$

Using Eq.(II) in Eq.(??) and then substituting Eq.(.), we find

$$E_p = -\alpha C \int dq \left[\frac{F_{123}(q)}{q^2 + 2m_b\omega_+(q)/\hbar} + \frac{F_{231}(q)}{q^2 + 2m_b\omega_-(q)/\hbar} + \frac{F_{312}(q)}{q^2 + 2m_b\Omega(q)/\hbar} \right] \quad (31)$$

where $C = 2m_b\omega_{LO}^2 R_p \Gamma[(d-1)/2]/\sqrt{\pi} \Gamma[d/2]$. The effective mass can be obtained as Taking the coefficient of k^2 term, the effective mass is obtained as

$$\frac{m}{m^*} = 1 + \alpha \frac{8m_b C}{\hbar^2 \sigma} \int dq q^2 \left[\frac{F_{123}(q)}{(q^2 + 2m_b\omega_+(q)/\hbar)^3} + \frac{F_{231}(q)}{(q^2 + 2m_b\omega_-(q)/\hbar)^3} + \frac{F_{312}(q)}{(q^2 + 2m_b\Omega(q)/\hbar)^3} \right] \quad (32)$$

V. RESULT AND DISCUSSION

The polaron binding energy and effective masses due to dynamic screening correction of electron-phonon interaction is calculated for several r_s values for dimensions $\beta=2, 2.5$ and 3 are shown in Table II. respectively. Here also the polaron properties are found to increase

with the increasing r_s and hence decrease with the increasing carrier density. This results due to the dynamic screening of the electron-phonon interaction is slightly greater than that of static screening.

The polaron properties due to dynamic screening also decrease as the dimension decreases which shows little higher values than static screening.

Here we have taken same parameters as in static screening such as the band mass $m_b=0.067m_0$, where m_0 is the electron mass, $\epsilon_\infty = 13.18$ and $\epsilon_0 = 10.89$. The value of $\alpha=0.067$ which is appropriate for calculating polaron properties in the weak coupling limit. The LO phonon energy ($\hbar\omega_{LO}$) is taken as 36.25 meV.

A. electron-electron interaction(plasmaron)

The self energy equation for the electron-electron interaction is given by

$$\Sigma_d(p, \epsilon k_m) = -\frac{1}{\beta} \sum_{m,q} V_{eff} G^0(\epsilon\omega n, \xi_{p+q})$$

Here $V_{eff} = \frac{V_o(q)}{\epsilon(q,\omega)}$ is the effective potential for electron-electron interaction term where $V_o(q)$ is the Fourier transform of Coulomb potential and $\epsilon(q,\omega)$ is the screened dielectric function.

$$\Sigma_{i,j}(p) = -\frac{V_0(q)}{\epsilon_\infty} \sum_{m,q} \left[\frac{n_F(\xi_{p+q}) ((\xi_{p+q} - \epsilon\omega_n)^2 + \tilde{\omega}_p^2 - \omega_p^2) - \tilde{\omega}_p^2 (n_B(\omega_p) + 1)}{2\omega_p(\xi_p - \xi_{p+q} - \hbar\omega)} - \frac{n_F(\xi_{p+q}) ((\xi_{p+q} - \epsilon\omega_n)^2 + \tilde{\omega}_p^2)}{2\omega_p(\xi_p - \xi_{p+q})} \right]$$

At zero temperature both n_F and n_B vanish and the self energy equation becomes

$$\Sigma_{i,j}(p) = -\frac{V_0(q)}{\epsilon_\infty} \sum_q \frac{\tilde{\omega}_p^2}{2\omega_p(\xi_p - \xi_{p+q} - \hbar\omega)}$$

The effective mass and binding energy equation are derived from the above self energy equation proceeding with eq(1) and (2).

$$E_p = -\alpha C \int dq \left[\frac{\tilde{\omega}_p^2}{2\omega_p(q^2 + 2m_b\Omega(q)/\hbar)} \right]$$

where $C = 2m_b\omega_{LO}^2 R_p \Gamma[(d-1)/2]/\sqrt{\pi} \Gamma[d/2]$. The effective mass can be obtained as Taking the coefficient of k^2 term, the effective mass is obtained as

$$\frac{m}{m^*} = 1 + \alpha \frac{8m_b C}{\hbar^2 \sigma} \int dq q^2 \left[\frac{\tilde{\omega}_p^2}{2\omega_p(q^2 + 2m_b\Omega(q)/\hbar)^3} \right]$$

VI. RESULTS AND DISCUSSIONS

The polaron binding energy and effective masses due to dynamic screening correction of electron-electron interaction is calculated for several r_s values for dimensions $\beta=2, 2.5$ and 3 are shown in Table III. Here also binding energies and effective masses are found to increase with the increasing r_s and hence decrease with the increasing carrier density. The polaron properties also decrease with increase in dimension.

Here we have taken same parameters as in previous such as the band mass $m_b=0.067m_0$, where m_0 is the electron mass, $\epsilon_\infty = 13.18$ and $\epsilon_0 = 10.89$. The value of $\alpha=0.067$ which is appropriate for calculating polaron properties in the weak coupling limit.

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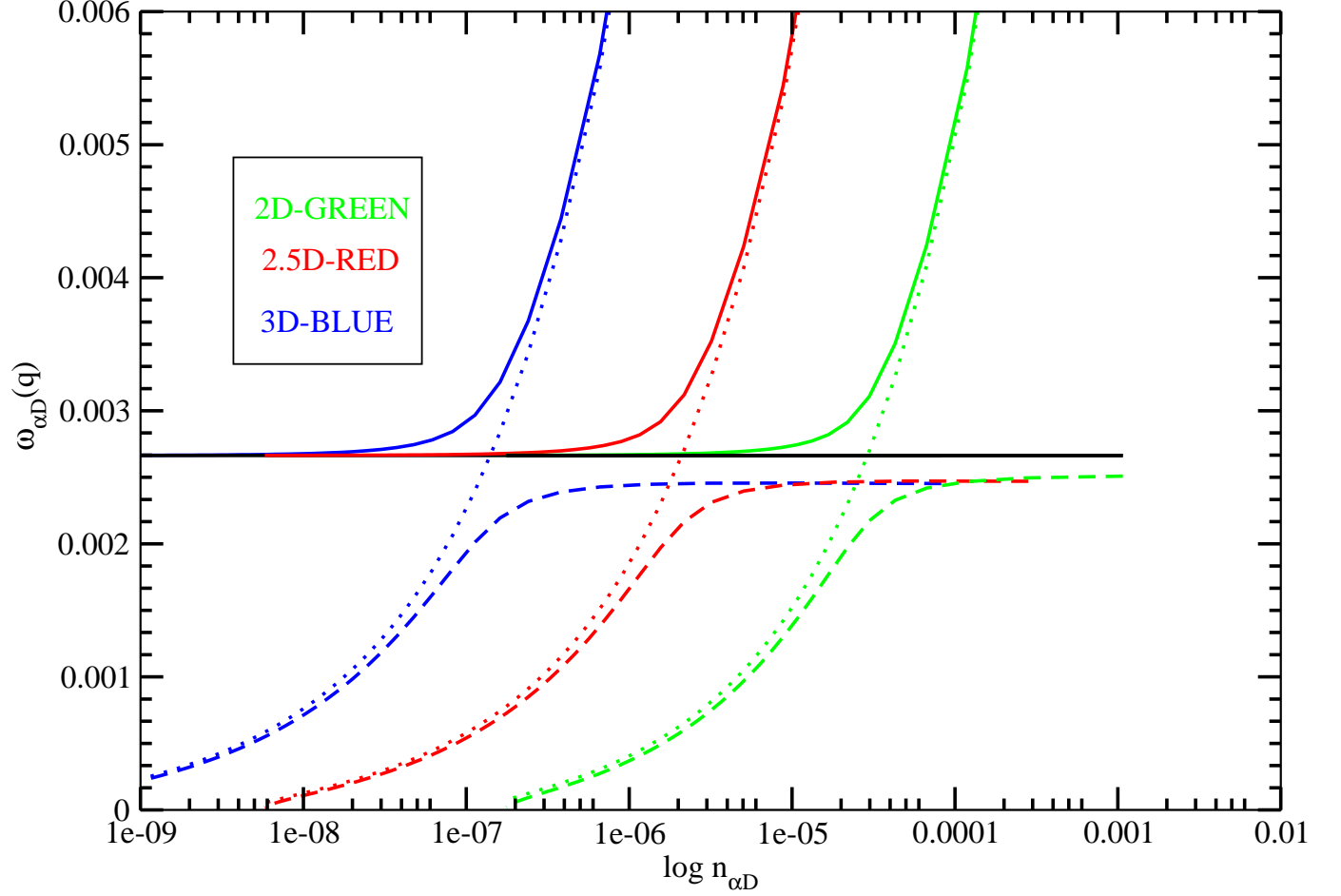


FIG. 1: Frequency of plasma modes as a function of logarithmic carrier density n_D in fractional dimension. ω_+ (solid line), ω_- (dashed line) and ω_p (dotted line) are shown for 2D, 2.5D and 3D in green, red and blue colour respectively. The black line shows the LO phonon frequency.

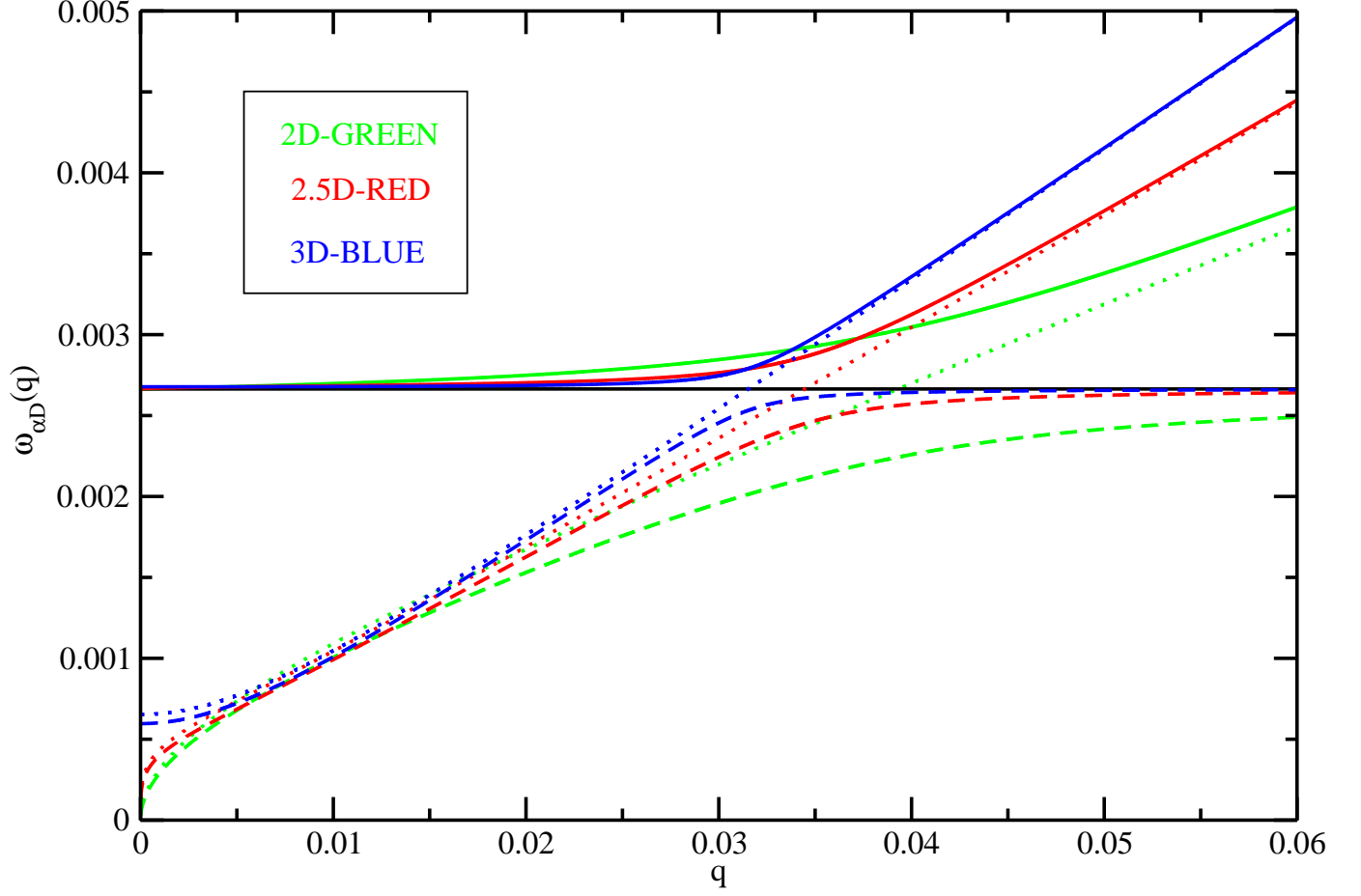


FIG. 2: Frequency of plasma modes as a function of wave vector q in fractional dimension. ω_+ (solid line), ω_- (dashed line) and ω_p (dotted line) are shown for 2D, 2.5D and 3D in green, red and blue colour respectively. The black line shows the LO phonon frequency.

r_s	Polaron Energy (eV)			Effective mass		
	$\beta = 2$	$\beta = 2.5$	$\beta = 3$	$\beta = 2$	$\beta = 2.5$	$\beta = 3$
0.001	0.416	0.296	0.210	1.120	1.067	1.041
0.01	0.417	0.315	0.255	1.121	1.070	1.047
0.10	0.424	0.325	0.272	1.123	1.071	1.049
1.0	0.430	0.328	0.274	1.124	1.072	1.050
10.0	0.430	0.340	0.275	1.125	1.073	1.051

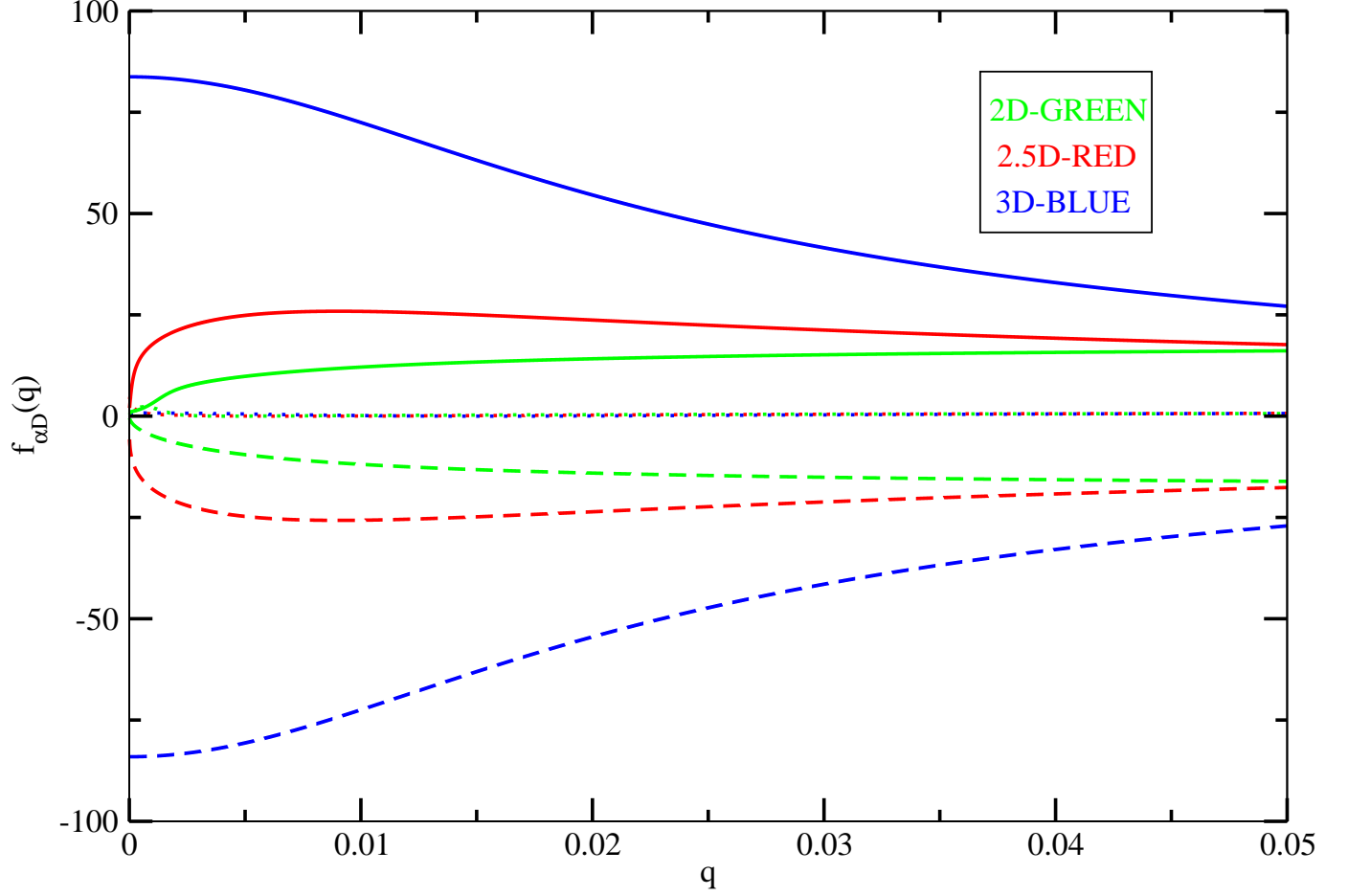


FIG. 3: The contribution factor $F(q)$ as a function of wave vector q in fractional dimension. F_+ (solid line), F_- (dashed line) and F_p (dotted line) are shown for 2D, 2.5D and 3D in green, red and blue colour respectively.

r_s	Polaron Energy (eV)			Effective mass		
	$\beta = 2$	$\beta = 2.5$	$\beta = 3$	$\beta = 2$	$\beta = 2.5$	$\beta = 3$
0.001	0.424	0.081	0.011	1.177	1.009	1.000
0.01	0.489	0.184	0.066	1.219	1.041	1.004
0.10	0.554	0.356	0.221	1.260	1.136	1.058
1.0	0.609	0.522	0.426	1.275	1.207	1.175
10.0	0.656	0.613	0.538	1.280	1.210	1.178

r_s	Polaron Energy (eV)			Effective mass		
	$\beta = 2$	$\beta = 2.5$	$\beta = 3$	$\beta = 2$	$\beta = 2.5$	$\beta=3$
0.01	0.019	0.002	0.001	1.000	1.000	1.000
0.05	0.094	0.011	0.004	1.010	1.005	1.003
0.10	0.189	0.024	0.008	1.075	1.041	1.020
0.5d0	0.934	0.139	0.037	6.263	3.970	3.067
1.0	1.858	0.304	0.067	26.968	13.995	9.801